Breakdown-free version of ILU factorization for nonsymmetric positive definite matrices

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In this paper a new ILU factorization preconditioner for solving large sparse linear systems by iterative methods is presented. The factorization which is based on A-biorthogonalization process is well defined for a general positive definite matrix. Numerical experiments illustrating the performance of the preconditioner are presented. A comparison with the well known preconditioner RIF p of Benzi and Tůma is also included. © 2009 Elsevier B.V. All rights reserved.

1. Introduction

In this paper we consider the solution of linear systems of the form

\[ Ax = b, \] (1)

where the coefficient matrix \( A \in \mathbb{R}^{n \times n} \) is large, sparse and nonsymmetric positive definite (NSPD), and \( b \) is a given right hand side vector using preconditioned conjugate gradient-type methods. Suppose that \( A \) admits the factorization

\[ A = LU, \] (2)

where \( L, U^T \) are unit lower triangular matrices and \( D \) is a diagonal matrix. If \( \tilde{L} \) and \( \tilde{U}^T \) are sparse unit lower triangular matrices approximating (in some sense) the matrices \( L \) and \( U^T \), respectively, and \( \tilde{D} \) is a nonsingular diagonal matrix approximating \( D \), then we say that matrix \( M \) with

\[ M = \tilde{L}\tilde{D}\tilde{U} \approx A, \] (3)

is an incomplete LU (ILU) factorization preconditioner for matrix \( A \). The transformed linear systems

\[ AM^{-1}u = b, \quad M^{-1}u = x, \] (4)

or

\[ M^{-1}Ax = M^{-1}b, \] (5)

have the same solution as system (1) and seem to be better-conditioned than the original system (1) to solve. It is well-known that an incomplete factorization of a general matrix \( A \) may fail due to the occurrence of zero pivots, regardless of