Three-Dimensional Boundary Element Method and Finite Element Method Simulations Applied to Stray Current Interference Problems. A Unique Coupling Mechanism That Takes the Best of Both Methods

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ABSTRACT

In this paper it will be demonstrated how a boundary element method (BEM) model based on so-called pipe elements and a finite element method (FEM) model that is limited in space can be coupled to study local effects on 3D structures in half space. The main idea is to replace the original 3D structure with an “equivalent pipeline network,” which is used in the BEM model to calculate the complete direct current (DC)-traction interference problem. In the FEM code on the other hand, an “equivalent bounding box” that surrounds the complete 3D structure is defined. The potential distribution on this bounding box is calculated using the BEM model and applied as boundary condition for the actual FEM calculation on the original 3D structure. This new approach is applied and validated on a typical underground car park geometry made of steel sheet walls that exhibits stray current interference from a neighboring DC-traction system. It has been demonstrated that this unique coupling technique works very well and allows the modeling of local effects on 3D structures influenced by large DC-traction systems.

KEY WORDS: boundary element method, direct current-traction, finite element method, pipe elements, stray current interference, underground car park

INTRODUCTION

Failures on underground metallic structures such as pipelines, storage tanks, and car parks can have severe environmental and/or economic consequences. Therefore, large investments have been made in studies on the corrosion prevention for buried structures. Important research is being conducted to determine and mitigate stray current interference, especially originating from direct current (DC)-traction systems. Moreover, because of the hidden character of the structure and their low accessibility, installation, survey, maintenance, and repair is intricate, elaborate, and expensive.

Numerical modeling can provide some relief by simplifying and optimizing installation, maintenance, and repair. When used in the planning phase, conceptual mistakes can already be traced before any actual installation, by calculating different setups in cheap, harmless, and fast simulations. Also, a model can provide reference values for measurements on operational sites that can help in the tracing and solving of any possible anomaly. Last but not least, the model technique creates a safe and cost-effective on-screen “virtual” test environment where new corrosion engineers can gain experience without long and expensive “trial and error” experiments on site.

In the past, several authors have applied both the boundary element method (BEM) and the finite element method (FEM) to a large range of corrosion or cathodic protection (CP)-related problems. Fu and Chan¹ used a FEM to model localized corrosion. Aoki, et al.,² analyzed current density and potential distributions using BEM. Adey³ used special BEM pipe elements to compute the performance of a sacrificial zinc anode system for the protection of a jacket-type offshore structure. Newman⁴ presented
CP design calculations based on an advanced model for the coating efficiency and the required protective current density. Brichau and Deconinck developed a BEM model that is particularly suited for large 3D networks of buried pipelines. This also includes the influence of stray currents originating from DC-traction systems (electrical train or tram vehicles), or other CP systems. Orazem, et al., demonstrated that a full 3D (multi and single domain) BEM approach is required to compute the protection level of large coating defects. They presented results for a pipe segment of limited length (10 ft [300 m]), in the presence of a parallel anode system. Kennelley, et al., developed a 2D FEM model to predict the current and potential distribution of an underground coated pipe in high-resistivity soil under an impressed current, parallel anode, CP system. The model was designed to study the effect of discrete holidays of various sizes on coated pipe without having to assume that holidays simply reduce the efficiency of the protective coating. The performance of the model was validated through comparison with experimental data. Riemer and Orazem developed a coupled BEM-FEM model for predicting the CP of pipeline networks, which was extended to treat CP of the bottoms of cylindrical aboveground storage tanks. Adey applied a full 3D approach to allow computation of the potential field in the neighborhood of jacket joints under the CP of sacrificial anodes. Rabiot, et al., applied the FEM to calculate the secondary potential and current density distribution at the surface of a buried tank. The model compared the relative influence of coating quality, electric conductivity of soil and position, and size and type of the sacrificial anodes (magnesium or zinc).

Purcar, et al., used a 3D BEM approach to determine the performance of a CP system applied to a buried pipeline of limited longitudinal dimension, surrounded by a U-shaped vault of infinite resistivity. Ridha, et al., developed a method to identify corrosion of steel in concrete by combining magnetic field measurement and a BEM inverse analysis.

In this work, both a BEM- and FEM-based software code will be used.

The BEM code is based on the mathematical model developed by Brichau, et al.,. This code is especially developed to calculate the DC-traction stray current interference on pipeline networks and is able to model the complete soil and the entire traction system consisting of rails, traction stations, overhead wires, and trains. The underground geometries that can be modeled, however, are limited to cylindrical structures (pipelines).

The FEM code, being fully integrated in the 3D CAD package SolidWorks, offers the possibility to model stray current interference on complex 3D structures. However, the computational domain in the CAD system is limited to a maximum of 1 km in each spatial dimension. This, together with the intrinsic space limitations of the FEM, makes it impossible to use it in its original form to model complete DC-traction interference problems.

Considering the power (and limitations) of the tools described above, it is clear that a combination of both would provide a very powerful simulation software that is able to calculate the interference from extended DC-traction systems on complex 3D structures such as underground car parks.

In this paper it will be demonstrated step-by-step how this can be achieved. The main idea is to replace the original 3D structure with an “equivalent pipeline network,” which is used in the BEM code to calculate the complete DC-traction interference problem. In the FEM code, on the other hand, an “equivalent bounding box” that surrounds the complete 3D structure is defined. The potential distribution on this bounding box is calculated using the BEM code and applied as the boundary condition for the actual FEM calculation on the original 3D structure.

Using this unique approach of coupling BEM and FEM it becomes possible to model the complete DC-traction problem directly in the FEM code. By using the bounding box approach, the computational domain for the FEM calculation will, when compared to standard FEM calculations, be very small, which considerably reduces both the grid generation and calculation time. In addition, the boundary conditions in the FEM model will be very simple, since the actual DC-traction system (or part of it) does not need to be modeled. As a consequence, also the number of nonlinear Newton-Raphson iterations goes down, since the potential distribution on the bounding box is no longer unknown, as is the case in standard FEM calculations.

In this section, a short overview of the equations to be solved and the boundary conditions applied will be given.

**Physical Model**

Due to the nature of the conductive medium and its large scale, no concentration gradients occur in the soil except for the well-known and very thin diffusion layer near the electrodes. Since the soil is assumed to feature only charge transport with normal ohmic resistivity effects, the potential model holds, being described by the Laplace equation:

\[ \nabla \cdot \mathbf{j} = 0 \quad \mathbf{j} = -\sigma \nabla U \quad (1) \]

The phenomena occurring in the diffusion layer and at the electrode interface are encompassed in the (nonlinear) boundary conditions described further on.
For domains with a constant electrical conductivity $\sigma$, Equation (1) simplifies to:

$$\nabla \cdot (\nabla U) = 0$$  \hspace{1cm} (2)

**Boundary Conditions**

For CP simulations, the steel electrode polarization is often encompassed in a single, measured electrode polarization curve, relative to a mixed corrosion potential $E_{corr}$:

$$j_n = f(V - U - E_{corr}) = f(\eta - E_{corr})$$  \hspace{1cm} (3)

where $U$ is the potential in the electrolyte adjacent to the electrode, $V$ is the metal potential, and $\eta(j_n)$ is the polarization overvoltage being a function of the local current density, $j_n$.

On insulation boundaries, the current density perpendicular to the surface should be zero, which results in the following boundary condition:

$$\mathbf{j}_n \times \mathbf{n} = j_n = -\sigma \nabla U \times \mathbf{n} = \sigma \mathbf{Q} = 0$$  \hspace{1cm} (4)

**Boundary Element Method**

When the BEM is applied, only the boundaries of the domain need to be discretized. The characteristic BEM equation for the contribution to node $i$ is:

$$cU^i + \int_{\Gamma} \frac{\partial w^*}{\partial n} d\Gamma = \int_{\Gamma} w^* \mathbf{Q} d\Gamma$$  \hspace{1cm} (5)

where $w^* = \frac{1}{4\pi r}$, the 3D Green function ($r$ representing the position vector) and $\mathbf{Q} = \frac{\partial U}{\partial n}$, the flux at the boundary nodes. $c^i$ is an integration constant for node $i$. $\Gamma$ is the 2D surface that encloses the 3D computational domain.

To apply BEM, the boundary $\Gamma$ is to be discretized into a series of $N$ nonoverlapping elements, transforming Equation (5) into:

$$c^i U^i + \sum_{j=1}^{N} \int_{\Gamma_j} \frac{\partial w^*}{\partial n} d\Gamma = \sum_{j=1}^{N} \int_{\Gamma_j} w^* \mathbf{Q} d\Gamma$$  \hspace{1cm} (6)

The index $j$ ranges over all elements of the domain and integration is performed over the surface $\Gamma_j$ of each element.

For the BEM method used in this paper, special “pipe elements” are used to perform the discretization. In addition, the ohmic drop along the pipeline axes (attenuation) is taken into account. Detailed information can be found elsewhere.

As a result, the final system of equations can be expressed in matrix form:

$$[H][U] = [G][J_n]$$  \hspace{1cm} (7)

with $[U]$ and $[J_n]$ unknown vectors of size $N$.

**Finite Element Method**

The FEM is a well-known numerical discretization technique. The method requires the discretization of the complete 3D domain. The domain $\Omega$ is divided in $N_e$ nonoverlapping elements $\Omega_e$, with $N_p$ as the total number of points. The FEM formulation supposes the approximation of the exact solution $U$ with a set of trial functions $U$. Shape functions $N_i$ are used to express this variation of $U$ over an element as:

$$\hat{U}^e = \sum_{i=1}^{N_e} N_i U_i$$  \hspace{1cm} (8)

where $\hat{U}^e$ is the trial function on element $e$, $U_i$ is the value of the unknown function $U$ in node $i$, and $N_e$ is the number of forming vertices for each element.

$$U = \sum_{i=1}^{N_p} N_i U_i$$  \hspace{1cm} (9)

By replacing $U$ from Equation (8) with Equation (9), the error (residual) $R$ results:

$$\nabla \cdot (-\sigma \nabla \hat{U}) = R$$  \hspace{1cm} (10)

The residual $R$ is distributed over the entire domain, so that the total error is minimized by averaging. The approximate solution of Equation (10) can be obtained from the Equation (11):

$$\int_{\Omega} W R d\Omega = 0$$  \hspace{1cm} (11)

where $W$ is the averaging function or weight function and $d\Omega$ is the unit volume.

In general, the use of trial functions also involves residuals (errors) in the boundary conditions, which can be written as:

$$\int_{\Gamma} W (\hat{U} - U) \frac{\partial U}{\partial n} dS_r + \int_{\Gamma} \left( \frac{\partial \hat{U}}{\partial n} - \frac{\partial U}{\partial n} \right) \mathbf{w} dS_r = 0$$  \hspace{1cm} (12)

Using partial integration and Gauss’s theorem of divergence, the general FEM expression is obtained:

$$\int_{\Omega} \nabla W \cdot \nabla \hat{U} d\Omega + \int_{\Gamma} \left( \hat{U} - U \right) \frac{\partial W}{\partial n} dS_r + \int_{\Gamma} \left( \frac{\partial \hat{U}}{\partial n} - \frac{\partial U}{\partial n} \right) \mathbf{w} dS_r = 0$$  \hspace{1cm} (13)

where the left-hand-side term gives the finite element discretization of the Laplace equation for the internal unknowns, while the right-hand-side term accounts for the boundary conditions and the finite element discretization of the Laplace equation on the boundary.
Different weighting functions may be chosen, but the most commonly used approach is known as Galerkin’s method, where one uses the same weighting functions as the global basis (shape) functions $N_i$.

As for the BEM method, this approach results in a general system of equations as given by Equation (14):

$$[K][U] = [F]$$  \hspace{1cm} (14)

where $[K]$ is the system matrix, $[U]$ is the vector of unknown potentials in the nodes, and $[F]$ is the vector with boundary conditions. The introduction of proper boundary conditions makes the system of equations unique.

**PROBLEM DESCRIPTION**

The geometry that will be investigated is a typical underground car park. The dimensions of the car park are 115 by 31.2 by 17 m (L/W/H). The top of the car park is 2 m below surface. Left of the car park, at a distance of about 20 m, is a railway system.

The main purpose of this study was to investigate the influence of DC-traction stray current interference on steel constructions. The questions that need to be answered are the following:

—Given the fact that the geometry is limited to pipes, can the BEM code be used to model DC-traction interference on real 3D structures using an equivalent pipeline network?

—Given the fact that the geometry is limited to 1 km in each spatial direction, can the FEM code be used to model DC-traction interference on real 3D structures using an equivalent bounding box?

To answer these questions, a step-by-step approach will be followed, including the following tasks:

—Compare the BEM and FEM codes when applied to cylindrical structures.

—Develop the equivalent BEM model.

—Evaluate the equivalent BEM model on a car park with “local” stray current interference.

—Evaluate the BEM/FEM coupling on cylindrical structures.

—Evaluate the BEM/FEM coupling on a car park under DC-traction stray current interference.

The problem depends on a lot of parameters, such as the resistivity of the soil, the polarization of bare steel, the electrical specification of the DC-traction system, and so on. In this feasibility study a uniform sandy soil with a resistivity of 200 $\Omega \cdot m$ will be taken. The polarization data as presented in Figure 1 was published previously.

**RESULTS**

**Boundary Element Method Model vs. Finite Element Method Model**

Before starting the development and evaluation of the approach as outlined above, a very short overview of the main differences between the two software tools that will be used is given:

**BEM model:**

—Only cylindrical structures can be modeled (use of “pipe elements” having uniformity along the circumference).

—Complete soil is modeled (semi-infinite domain with earth surface level considered as insulator).

—Zero volt reference potential is imposed at infinity (“far field”).

**FEM model:**

—All structures can be modeled.

—Only part of soil can be modeled (finite domain due to the CAD system size limitation (1 km) and FEM approach).

—Zero volt reference potential is imposed at structure.

**Task 1: Comparison of Boundary Element Method and Finite Element Method when Applied to Cylinders**

Before going into detail on how the equivalent pipeline model is created in the BEM code, it will be
demonstrated that both codes, although based on completely different methods and implementation techniques, give the same results when applied to cylindrical geometries. For that purpose a simple geometry with two vertical bare pipes (full cylindrical rods) will be modeled.

The pipes have a length of 20 m and are buried 2 m below surface. The pipes lie at a distance of 20 m in both X and Y directions and are connected to each other via a voltage of 1 V. This creates a cathode-anode system with the cathode connected to the more negative pole of the voltage supply. Around the cathode, the soil potential will exhibit a more negative potential with respect to the soil potential near the anode.

**Boundary Element Method Simulations** — The calculated solver output is presented in Table 1. The total current output is about 93 mA with a negative value at the cathode and a positive value at the anode. The average current density is about 1.48 mA/m².

Figure 2 presents the calculated earth surface potential in the region near the cathode-anode system. As already mentioned, the zero reference potential in the BEM model is imposed at infinite (far field), giving an earth surface potential distribution that is almost symmetric around zero, except for some small deviations due to the nonlinear behavior of the bare steel polarization.

It can be observed by the “distorted” (noncylindrical) potential lines in the region between both electrodes that they are in each others’ field of influence.

This is not surprising considering the very small distance between both bare steel cylinders.

**Finite Element Method Simulations** — As already mentioned, the BEM model is based on a semi-infinite domain with the earth surface level considered as an insulator, which means that only the pipes need to be discretized. The FEM model on the other hand requires a bounding box (with the upper face at the earth surface level) to “close” the computational domain. The faces of this bounding box are assumed to be an insulator, which implies that no current can enter or leave the box. This condition is implicitly fulfilled for the upper face. For the lateral and lower faces, however, this assumption is only justified provided that the box is big enough. This requires that these faces are in the “far field,” being the region out-

| Time needed to Construct System Matrix | 0 seconds |
| Iteration Number 1 | : Residual = 1.677E–03 |
| Time needed to Solve System of Equations | 0 seconds |
| Iteration Number 2 | : Residual = 2.571E–06 |
| Time needed to Solve System of Equations | 0 seconds |
| Vinf = 0 | 

####### Outgoing Current per Branch #######
Cathode : I = –9.305E+01[mA], J = –1.481E+00[mA/m²]
Anode : I = +9.306E+01[mA], J = +1.481E+00[mA/m²]

Current Balance of All Branches = +4.792E–03 [mA]

Total Time needed for the Solution Process = 1 seconds
Aside the influence zone from both pipes (characterized by a constant soil potential). The region where the far field starts strongly depends on the soil resistivity.

To make sure that the bounding box is indeed at far field, several FEM simulations have been done with increasing size of the bounding box. For each simulation the minimum and maximum values for the calculated potential at the lateral and lower faces of the bounding box are compared. When the difference between both values was less than 10 mV, with 1% being the imposed potential difference between both pipelines, the bounding was considered to be far field.

For the current simulation it turns out that the bounding box needs to be at least 420 m in both X and Y directions and 200 m in the Z direction to fulfill the far-field condition. This is roughly about 20 times the dimension of the actual cathode-anode configuration.

The calculated FEM potential distribution on the lateral and lower faces is presented in Figure 3 (upper face not presented). The minimum and maximum potentials are 1.15 V and 1.16 V, respectively. This potential distribution is calculated with the zero volt reference potential imposed at the cathode and a corrosion potential of –0.66 V is taken into account at both electrodes.

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>Finite Element Method Solver Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result per body...</td>
<td>I [mA]</td>
</tr>
<tr>
<td>Anode</td>
<td>94.291000</td>
</tr>
<tr>
<td>Box</td>
<td>0.000000</td>
</tr>
<tr>
<td>Cathode</td>
<td>–94.291000</td>
</tr>
<tr>
<td>Balance</td>
<td>0.000000</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>ELECTRODE SYSTEM : Anode [non-resistive]</td>
<td>TOTAL CURRENT : 94.291000 [mA] TOTAL SURFACE : 64300728 [m²]</td>
</tr>
<tr>
<td>Quantity</td>
<td>Average</td>
</tr>
<tr>
<td>U [V] :</td>
<td>1.626972</td>
</tr>
<tr>
<td>J [mA/m²] :</td>
<td>1.466414</td>
</tr>
<tr>
<td>UOff [V] :</td>
<td>–0.626972</td>
</tr>
<tr>
<td>V [V] :</td>
<td>1.000000</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>ELECTRODE SYSTEM : Cathode [non-resistive]</td>
<td>TOTAL CURRENT : –94.291000 [A] TOTAL SURFACE : 64299908 [m²]</td>
</tr>
<tr>
<td>Quantity</td>
<td>Average</td>
</tr>
<tr>
<td>U [V] :</td>
<td>0.680964</td>
</tr>
<tr>
<td>J [mA/m²] :</td>
<td>–1.466433</td>
</tr>
<tr>
<td>UOff [V] :</td>
<td>–0.680964</td>
</tr>
<tr>
<td>V [V] :</td>
<td>0.000000</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>EXTERNAL CONNECTIONS :</td>
<td>Label V1 [V] V2 [V] I12 [mA]</td>
</tr>
<tr>
<td>VG1</td>
<td>0.000000</td>
</tr>
<tr>
<td>GR1</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Total calculation time = 20 [sec]

---

**FIGURE 3. FEM potential distribution on lateral and lower faces.**
Comparison Between Boundary Element Method and Finite Element Method — Table 3 gives an overview of the average calculated values using both codes (only values on the anode are presented).

From this table it can be seen that the agreement between the BEM and FEM codes is very good. Notice that the average FEM current density is a little bit smaller than that presented by the BEM code while the corresponding total current is larger. This “contradiction” is due to the fact that in the FEM model the top and bottom disks of the cylinder are taken into account too (hence, increasing the total surface), while this is not the case in the BEM model.

Figures 4 and 5 give a comparison between the calculated BEM and FEM current densities along the cathode and anode, respectively. The FEM values are average radial values along the hull of the pipe, while in the BEM code (due to the uniformity as imposed by the model) the radial values are constant anyway.

From these graphs it can clearly be observed that both software tools, although based on totally different techniques and implementations, give almost the same results (when applied to identical geometries). At the end of the pipes there is a difference due to the fact that in the BEM code the effect of the disk at the end of the pipe is not taken into account.

In the next paragraph it will be demonstrated how the BEM code can still be used successfully when modeling 3D structures by applying an equivalent pipeline model.

Task 2: Developing the Equivalent Boundary Element Method Model

To be able to model 3D structures (such as car parks) using the BEM code, the original geometry

<table>
<thead>
<tr>
<th>Quantity</th>
<th>BEM</th>
<th>FEM</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total current (mA)</td>
<td>93.05</td>
<td>94.29</td>
<td>1.33</td>
</tr>
<tr>
<td>Average current density (mA/m²)</td>
<td>1.48</td>
<td>1.47</td>
<td>0.68</td>
</tr>
</tbody>
</table>

FIGURE 4. Current density at the cathode. Comparison between BEM and FEM.

FIGURE 5. Current density at the anode. Comparison between BEM and FEM.
needs to be replaced with a set of pipelines that “be-

have in the same way as the original geometry.”

The development of a sound set of formulas,

which transforms a 3D structure into a network of

pipelines that has a similar stray current pattern,
cannot be done without a fundamental mathematical

study. Parameters that need to be investigated are the

soil resistivity, the polarization, the pipeline size (in-

ner/outer diameter), the inner-pipeline distance, and

the number of pipelines.

Such a fundamental mathematical study, how-

ever, is outside the scope of this project. Instead, a

more pragmatic approach has been followed. The key

idea is to keep the model as simple as possible and

easy to use. Considering this, it has been decided to

to represent the car park by a set of horizontal, closed

pipes as presented in Figure 6. To ensure electrical

continuity, each pipeline is directly connected to his

lower neighbor using a direct bond (nonresistive con-

nection).

In the equivalent BEM model, the interior volume

of the car park is filled with soil. This does not con-

tribute any substantial error to the model because of

the enormous conductivity difference between the soil

and the pipe steel, which is assumed to be a perfect

conductor.

To keep the model as simple as possible, it is as-

sumed that all pipes have the same diameter (D) and

that the distance (d) between two neighboring pipes is

equal. It is also assumed that the top (bottom) of the
car park coincides with the top (bottom) of the first
(last) pipeline.

Figure 7 shows a scheme of the equivalent BEM

model using a layer of horizontal pipes with all rel-

vant parameters.

From this scheme it is clear that for a given

height H of the car park, the following three param-

eters need to be determined: D, d, and N. The param-

eters are bounded by the following two relations:

—The pipes need to cover the total height:

\[ \frac{D}{2} + (N - 1)d + \frac{D}{2} = H \]  

(15)

—The total pipe area needs to be the same as the

structure area (per unit length):

\[ N\pi D = H \]  

(16)

Since there are only two relations to determine three

parameters, one of them can be chosen indepen-

dently. Table 4 gives the values for the pipe diameter

(D) and pipe distance (d) based on the number of hori-

zontal pipes. Simulations will be done for three differ-

ent N values, being 5, 10, and 20.

**Task 3: Evaluating the Equivalent Boundary Element Method Model**

To access the validity and accuracy of the pro-

posed equivalent model, the original car park con-

figuration with DC-traction system will be simplified.

This way, secondary effects that make the comparison

between the two software tools unnecessarily difficult

are reduced to a minimum. The car park itself is pre-
sented by a set of planar faces (a box) with no internal resistance, while the DC-traction system is replaced by a cathode-anode system. Although not the same, this system will also introduce a stray current pattern of currents that enter the car park somewhere to discharge at another location as the real DC-traction system does.

The car park is represented by a box with dimensions 115 by 31.2 by 17 m (L/W/H), 2 m below the surface. The top and bottom face of the car park are insulators. On the left of the car park, at a distance of 20 m, is a cathode-anode system consisting of two vertical pipes, connected to each other via a voltage of 50 V. The pipes have a length of 10 m and are buried 2 m below surface. The distance between the pipes is 31.2 m so that the center of both pipes coincides with the corners of the car park.

Finite Element Method Simulations — The bounding box of the FEM simulation is 800 by 700 by 400 m. The maximum potential difference on the lateral and lower faces of the bounding box is 75 mV, which is only a fraction of the total imposed voltage of 50 V, indicating that the bounding box indeed is big enough to be considered far field.

Figure 8 presents the calculated current density distribution at the car park near the cathode-anode system. From this picture the stray current pattern can be seen clearly. At the anode, introducing a positive soil potential, the current is discharged and picked up by the car park (CP). The current that enters the structure, of course, has to go back to the cathode and will leave the car park in the region near the cathode. The zero current density line is somewhere in the middle of the face opposite to the cathode-anode system (left side). The front, back, and right side of the car park, being more remote with respect to both the cathode and anode, are hardly influenced.

Boundary Element Method Simulations — Figure 9 presents the results on the upper pipeline (z = -2.27 m) for an equivalent model with N = 10. It is clear that the same stray current pattern as for the FEM code is obtained.

Notice that in the BEM model, since we are using layers of closed pipelines, beginning and end points of each pipeline have the same coordinates. The numbering of the relevant geometrical points (from 1 to 6) for each pipeline starts in the middle of the right side of the car park (X = 115, Y = 15.6 m) and is done in the clockwise direction. This implies that the left side of the car park, being the section between points 3 (X = Y = 0 m) and 4 (X = 0, Y = 31.2 m), is faced directly toward the cathode-anode system. Points 1 and 6 have the same geometrical location.

Comparison Between Finite Element Method and Boundary Element Method — Figures 10 and 11 show the comparison between the FEM and BEM codes using an equivalent model, where N = 10. Results have been presented at depths z = -2.27 (top of the car park, i.e., the first pipeline in the BEM model) and z = -9.58 (center of the car park, i.e., 5th pipeline in the BEM model). The FEM results are obtained by extracting the calculated results along the lines with a constant z value. These lines follow the contour of the
box, starting from point 1 and ending at point 6 in Figure 9.

From this graph it can be seen clearly that there is an excellent agreement between the calculations of both software tools. Results are presented in a region of 60 m (from $L = 100$ to 160 m) near the cathode-anode system. In the other regions, the current density is very low, since there is hardly any interference.

Figure 12 shows the calculated earth surface potential near the cathode-anode system as obtained by the FEM and BEM codes ($N = 10$). The zero reference potential for the FEM model has been shifted to the far field to be able to compare with the BEM results. Again, an excellent agreement is obtained.

Task 4: Evaluating the Boundary Element Method/Finite Element Method Coupling on Cylindrical Structures

In the previous sections, it has been outlined how the BEM code can be used to model 3D structures by using an equivalent pipeline network. It has been proven that the equivalent model with 10 pipelines ($N = 10$) gives excellent results when compared to FEM simulations obtained for a car park model with a simplified stray current interference system.

Given the fact that the BEM code models the complete semi-infinite domain, it was necessary in the FEM model to make the bounding box big enough to
ensure far-field conditions on the lateral and bottom faces, which are considered to be insulators.

The obvious questions are:
—Given the fact that the bounding box is too small, can the FEM code still give correct results?
—Given the fact that the geometry in the CAD system is limited to 1 km in each direction, can the FEM code be used to model DC-traction interference on real 3D structures?

To answer the first question, we will recall the simple geometry with two vertical, bare pipes as previously used. At the time, a very large bounding box (420 by 420 by 200 m) had to be taken to ensure far-field conditions on the lateral and bottom faces. Here, we will investigate what happens if the bounding box is limited to 60 by 60 by 40 m, yielding a volume reduction with a factor of 250.

The calculated FEM current using this small bounding box is 87.4 mA, roughly 10% lower than the 94 mA value obtained using the large bounding box. The reason for this is quite simple. Since we reduce the size of the bounding box, both pipes are relatively close to the lateral and bottom faces (being insulators). Therefore, the current exhibits an increased resistance when compared to the large bounding box. This increased resistance results in a reduced current, since we are working with a constant impressed voltage.

Figure 13(a) presents the FEM earth surface potential for the small bounding box. The effect of the box can be seen immediately. The potential lines need to be perpendicular to the walls of the box, since they are insulators. This strongly “distorts” the electrical potential distribution in the region near the box.
However, this bounding box, although being too small, can still yield excellent results, which will become clear very soon. The idea is the following:

—Use the BEM solver to calculate the potential distribution on the structure (this has already been done).
—Use the BEM solver to calculate the potential distribution on the lateral and bottom faces of the small bounding box, which has been defined in the FEM code.
—Use the potential distribution on the lateral and bottom faces of the small bounding box as calculated by the BEM solver as the boundary condition for the FEM calculation.

When re-running the FEM simulation, following the approach as outlined above, the obtained current is about 94 mA, which is the same value as obtained using the original FEM model with the large bounding box. The same observations can be made when looking at the earth surface potential distribution calculated using this coupling mechanism (Figure 13[b]). The potential lines are no longer distorted and are identical to the ones originally obtained.

Notice that the potential distribution on the upper face of the bounding box (the face presented in Figure 13) is calculated in the FEM code. This face could have been added to the coupling mechanism, i.e., added to the faces for which the potential is calculated in the BEM code. It has been opted not to add this face to test the coupling mechanism as much as possible.

**Task 5: Evaluating the Boundary Element Method/Finite Element Method Coupling Under Direct Current-Traction Interference**

The BEM/FEM coupling mechanism as outlined in the previous section will now be used to model large DC-traction interference problems in the FEM code. Consider the example as outlined in Figure 14.

The car park as modeled before is influenced by a DC-traction system that resides at a distance of 20 m. At opposite sides of the track, there are two traction stations at a distance of 10 km that deliver 1,500 V each. The tracks have a resistance to earth of 1,000 Ω·m². The cross section of the overhead wires is 1.5 cm² with a specific resistance of 1.78 × 10⁻⁸ Ω·m. A train with an internal resistance of 0.2 Ω is situated 50 m away from the beginning of the car park. The bounding box for the FEM simulations is 335 by 230 by 100 m.

**Boundary Element Method Simulations** — First, the BEM code has been used to calculate the influence of the DC-traction system on the equivalent car park geometry. These kinds of influences can easily be modeled in the BEM code, since the complete half-space is taken into account. This implies that all systems that are in contact with the soil automatically interfere with each other.⁵⁻⁶,¹⁵⁻¹⁷

Figure 15 presents the calculated metal potential along the track. The train, located near the car park, receives a current from both traction stations and sends it back via the rails, except, of course, for the stray current that leaks into the soil due to the limited track-to-earth transition resistance. At the position of the train, the potential reaches its maximum of about 7.5 V, while at both traction stations the potential drops to ~8.1 V.

Figure 16 presents the results on the upper pipeline (z = −2.27 m) for an equivalent model with N = 10. From this figure the stray current pattern can clearly be observed. Due to the high track potential at the location of the train, the left part of the car park, being closest to the train, receives the stray current. The highest current density values are found near the corners of the car park due to the edge effects. The current travels along the car park and discharges at remote areas from the train to go back to the traction stations. The right part of the car park has the highest anodic current, especially near the corners where the corrosion current is the highest.

In Figure 17, the calculated current density on the upper, middle, and lower pipelines are presented. From this overview, it can be noticed that on the left side of the park most of the stray current is picked up
**FIGURE 15.** BEM calculated track potential.

**FIGURE 16.** BEM current density on car park with DC-traction interference.

**FIGURE 17.** BEM current density along car park as a function of depth.
by the upper pipeline, being closest to the track. The lower part of the left side gets more current than the middle part due to the edge effect (reduced resistance toward the lower part). At the other side of the car park where the current discharges, the situation is different. Here, most of the current is discharged from the lower part of the car park.

Finite Element Method Simulations — As before, the BEM solver has been used to calculate the potential distribution on the bounding box defined in the FEM code. In this case, however, the situation is somewhat different, since the upper face of the bounding box is also passed to the BEM solver. This way we do not need to introduce the actual track geometry and equivalent electrical parameters, which can be a cumbersome task, especially since we are only modeling a very small section of the actual track (230 m instead of 10 km).

Figure 18(a) shows the surface mesh on the bounding box created in the FEM code. The mesh has been refined in the region near the track, since at that location the largest potential gradients in the soil occur, as can be seen easily from Figure 18(b). Notice that the highest earth potential, being 5.5 V, obtained at the location of the train, is about 2.0 V lower than the corresponding track potential, as presented in Figure 17. This is due to the overpotential reaction that takes place at the rail/earth interface.

The bounding box potential as calculated by the BEM solver is used again as the input boundary condition for the FEM simulation. As outlined above, the complete bounding box has been passed to the BEM solver. Therefore, the FEM configuration is very simple, only the car park and bounding box are taken into account; there is no need to model the track (including rails, overhead wires, trains, and traction stations) because their influence is taken into account via the boundary conditions. Figure 19 shows the calculated current density on the car park. Exactly the same stray current pattern as obtained using the BEM solver can be observed.

Comparison Between Boundary Element Method and Finite Element Method — Figures 20 and 21 show the comparison between the BEM (model with 10 pipes) and FEM solvers as calculated above. Results have been presented at depths \( z = -2.27 \) (top of the car park, i.e., first pipeline in the BEM model) and \( z = -9.58 \) (center of the car park, i.e., 5th pipeline in the BEM model). From these graphs it can be seen that, as before, there is a very good agreement between both software tools.

From these simulations it is clear that the coupling of BEM and FEM is very powerful, especially when modeling large models (e.g., when DC-traction is involved) that cannot be taken into account as such because of the size limitations of the CAD system and the intrinsic limitations of the FEM.

CONCLUSIONS

❖ In this paper, a unique coupling mechanism has been proposed between an existing BEM and FEM solver that allows the study of local effects on 3D structures in half-space.
❖ The BEM solver is developed especially to calculate the DC-traction stray current interference on pipeline networks. The geometries that can be modeled in the BEM code are limited, however, to cylindrical structures (pipelines). The FEM solver, on the other hand, offers the possibility to model stray current...
interference on complex 3D structures but is limited in the size of models that can be handled. Considering the power (and limitations) of both tools, it has been demonstrated in a step-by-step approach that a combination of both solvers provides a very powerful simulation software that is able to calculate the interference from extended DC-traction systems on 3D structures such as car parks.

❖ It has been demonstrated that both packages give exactly the same results when applied to cylindrical structures. Based on this observation, an equivalent BEM model that replaces the car park has been developed and validated using a simple cathode-anode interference system. This validation shows that the BEM solver is perfectly capable of modeling car park geometries, using a simple equivalent model based on layers of horizontal pipelines.

❖ To apply the FEM solver to extended DC-traction interference simulations, a BEM/FEM coupling mechanism has been proposed. The main idea is to use the “equivalent pipeline network” approach validated above and calculate the potential distribution on an equivalent FEM bounding box in the BEM code. This is then used as a boundary condition for the actual FEM calculation on the original 3D structure.

❖ This approach has been tested on the simple cathode-anode example used before, yielding excellent results. In a second step the same approach has been used successfully to deal with large DC-traction interference problems. By using this bounding
box approach, the computational domain in the FEM solver is, when compared to standard FEM calculations, very small, which considerably reduced both the grid generation and calculation time. In addition, the boundary conditions in the FEM solver are very simple, since the actual DC-traction system (or part of it) does not need to be modeled. As a consequence, the number of nonlinear Newton-Raphson iterations goes down drastically, since the potential distribution on the bounding box is no longer unknown, as is the case in standard FEM calculations.

REFERENCES